sum of the same odd powers of two integers, it is factorable); m can thus be only an integral power of 2; as, 1, 2, 4, 8, ,$\cdots$. This means that $\beta = 2^n + 1$, where n must be such a positive integer that $p = 2^{2^n} + 1$ is a prime.

Summarizing, we have for solutions of the given equation $\phi(p^\alpha) = \phi(q^\beta)$:

Case (1), $p = q$. Any prime, even or odd, is a root, if $\alpha = \beta$.

Case (2), $p \neq q$. One root is the even prime 2 with any exponent 2^n such that $2^{2^n} + 1$ is a prime, this odd prime being the other root for this individual solution. In this case the exponent of the odd prime is unity, while that of the even prime is of the form 2^n, as just indicated.

It may be added that the general form for n such that $2^{2^n} + 1$ is always a prime is still undetermined. It is of interest to note that this form is one which Fermat conjectured was always a prime for positive integral values of n. While this is true for many values of n, as 0, 1, 2, 3, $\cdots$, the factors have been found for other values of n: thus $2^{2^n} + 1$ is composite for $n = 5, 6, 9, 11, 12, 18, 23, 36, 38$.[1] These numbers are too large to be handled readily (or at all) when expressed in ordinary Arabic notation. The French encyclopedia of mathematics (t. 1: 3, p. 51) states that $2^{2^{36}}$ is an integer of more than twenty billion digits. The two following interesting propositions have been proposed, but no published proofs seem to exist: All the numbers represented in the series $2 + 1$, $2^2 + 1$, $2^{2^2} + 1$, $2^{2^{2^2}} + 1$, $\cdots$ are primes. The second is due to Eisenstein: There is an infinity[2] of prime integers of the form $2^{2^n} + 1$.

186. Proposed by H. PRIME, Boston, Mass.

Show that $\dfrac{(n+1)(n+2)\cdots(2n-2)}{(n-1)!}$ is an integer for all values of n.

SOLUTION BY THOMAS E. MASON, Bloomington, Ind.

The expression in the problem is equivalent to (1) $(2n - 2)!/n!(n - 1)!$ and the expression (2) $(m + n)!/m!n!$ is an integer, since it is a coefficient in the binomial expansion for the exponent $m + n$.

Each prime factor of the denominator of (1) which is not a factor of n occurs in the denominator as many times as it occurs in $(n - 1)!(n - 1)!$, and therefore occurs at least as many times in the numerator, since by (2) $(2n - 2)!/[(n - 1)!(n - 1)!]$ is an integer.

Any prime factor of n (different from 1) is not contained in $n - 1$, and therefore occurs in the denominator of (1) as many times as it occurs in $n!(n - 2)!$. It occurs at least as many times in the numerator since by (2) $(2n - 2)!/n!(n - 2)!$ is an integer.

The expression in the problem is therefore an integer for all values of n since the numerator contains every prime factor of the denominator at least as many times as it occurs in the denominator.

Also solved by B. F. YANNEY and H. C. FEEMSTER.

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[1] Intermed. Math., 10 (1903), p. 158; 11 (1904), p. 79. Lucas, Théorie des Nombres, p. 51.

[2] Lucas, Théorie des Nombres, p. 355.